Chapter 12 Pre-Algebra Surface Area and Volume

Name ____________________

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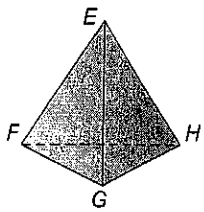
12-1 Study Guide and Intervention

Three-Dimensional Figures

Identify Three-Dimensional Figures A **prism** is a polyhedron with two parallel, congruent **bases**. A **pyramid** is a polyhedron with one base. Prisms and pyramids are named by the shape of their bases, such as triangular or rectangular.

Example 1 **Identify the figure. Name the bases, faces, edges, and vertices.**



This figure has one triangular base, $\triangle FGH$,

so it is a triangular pyramid.

faces: *EFG, EGH, EFH, FGH*

edges: $\overline{EF}$, $\overline{EG}$, $\overline{EH}$, $\overline{FG}$, $\overline{FH}$, $\overline{GH}$

vertices: *E, F, G, H*

Example 2 **Identify the figure. Name the bases, faces, edges, and vertices.**

This figure has two circular bases, *A* and *B*,

so it is a cylinder.

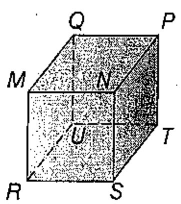
faces: *A* and *B*

The figure has no edges and no vertices.

Exercises

Identify each figure. Name the bases, faces, edges, and vertices.

1.



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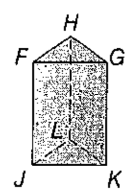

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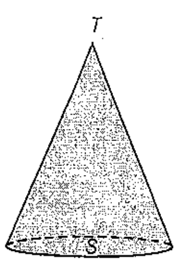

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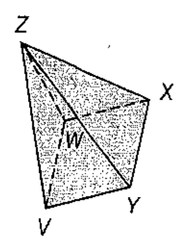

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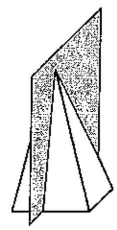
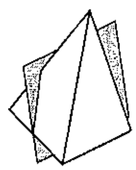
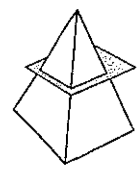
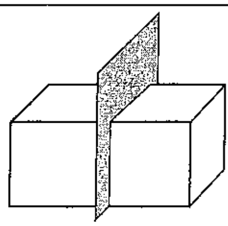
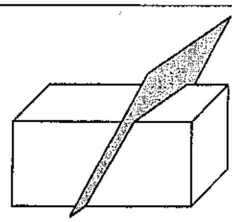
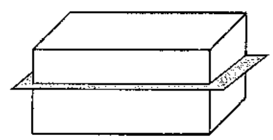
12-1 Study Guide and Intervention *(continued)*

Three-Dimensional Figures

Cross Sections When a plane intersects, or slices, a figure, the resulting figure is called a **cross section**. Figures can be sliced vertically, horizontally, or at an angle.

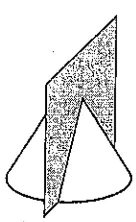
Vertical Slice	Angled Slice	Horizontal Slice
		
This cross section is a triangle.	This cross section is a trapezoid.	This cross section is a square.

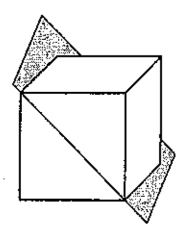
Example **Draw and describe the shape resulting from the following vertical, angled, and horizontal cross sections of a rectangular prism.**

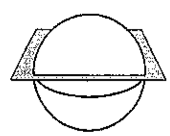
Vertical Slice	Angled Slice	Horizontal Slice
		
This cross section is a rectangle.	This cross section is a parallelogram.	This cross section is a rectangle.

Exercises

Draw and describe the shape resulting from each cross section.

1. 

2. 

3. 

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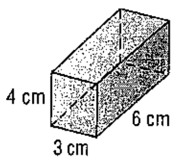
12-2 Study Guide and Intervention

Volume of Prisms

Volume of Prisms To find the volume V of a prism, use the formula $V = Bh$, where B is the area of the base, and h is the height of the solid.

Example **Find the volume of each prism.**

a.



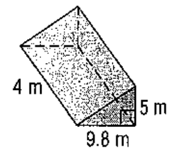
$V = Bh$

$V = (3 \cdot 6)4$

$V = 72$

The volume is 72 cm^3.

b.



$V = Bh$

$V = \left(\frac{1}{2} \cdot 9.8 \cdot 5\right)4$

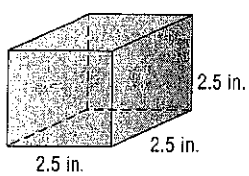
$V = 98$

The volume is 98 m^3.

Exercises

Find the volume of each figure. If necessary, round to the nearest tenth.

1.



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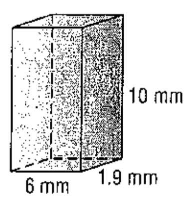
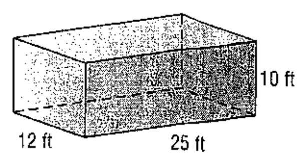


3.



4. Rectangular prism: length 9 millimeters, width 8.2 millimeters, height 5 millimeters

5. Triangular prism: base of triangle 5.8 feet, height of triangle 5.2 feet, height of prism 6 feet

6. Find the width of a rectangular prism with a length of 9 inches, a height of 6 inches, and a volume of 216 cubic inches.

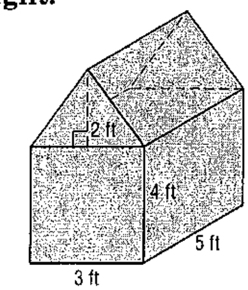
7. Find the base length of a triangular prism with a triangle height of 8 feet, a prism height of 7 feet, and a volume of 140 cubic feet.

12-2 Study Guide and Intervention *(continued)*

Volume of Prisms

Volume of Composite Figures Figures that are made up of more than one type of figure are called composite figures. You can find the volume of a composite figure by breaking it into smaller components. Then, find the volume of each component and finally add the volumes of the components to find the total volume.

Example TOYS **Find the volume of the play tent at the right.**



The figure is made up of a rectangular prism and a triangular prism. The volume of the figure is the sum of both volumes.

$V(\text{figure}) = V(\text{triangular prism}) + V(\text{rectangular prism})$

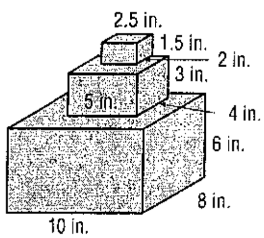
$V(\text{figure}) = Bh + \ell wh$ Write the formulas for the volumes of the prisms.

$= \frac{1}{2} \cdot 3 \cdot 2 \cdot 5 + 4 \cdot 3 \cdot 5$ Substitute the appropriate values.

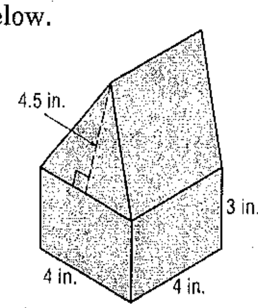
$= 15 + 60 \text{ or } 75 \text{ ft}^3$ Simplify.

Exercises

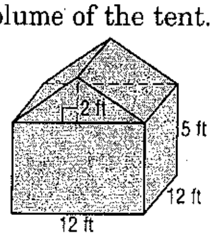
1. **GIFTS** Jamie made the tower of gifts shown below. Find the volume of the gifts.



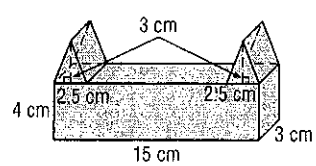
2. **GEOMETRY** Find the volume of the figure below.



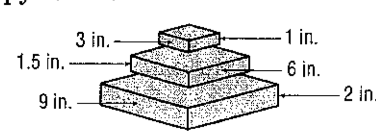
3. **TENTS** Mrs. Lyndon bought a patio tent. Find the volume of the tent.



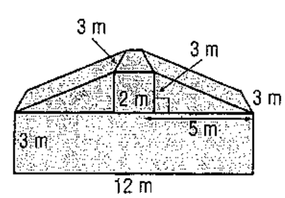
4. **MOLDS** Find the volume of the sandcastle mold shown below.



5. **PYRAMIDS** Ricky built a model of a square step pyramid. Find the volume of the pyramid.



6. **CANOPIES** Find the volume enclosed by the canopy shown below.



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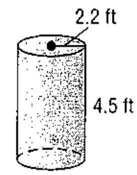
12-3 Study Guide and Intervention

Volume of Cylinders

Volumes of Cylinders Just as with prisms, the volume of a cylinder is based on finding the product of the area of the base and the height. The volume V of a cylinder with radius r is the area of the base, πr^2, times the height h, or $V = \pi r^2 h$.

Example 1 **Find the volume of the cylinder.**



$V = Bh$	Volume of a cylinder.
$V = \pi r^2 h$	Replace B with πr^2.
$\approx 3.14 \cdot 2.2^2 \cdot 4.5$	Replace π with 3.14, r with 2.2, and h with 4.5.
≈ 68.4	Simplify.

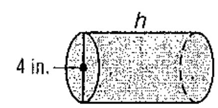
The volume is about 68.4 cubic feet.

Check: You can estimate to check your work.

$V = \pi r^2 h \approx 3 \cdot 2^2 \cdot 5$	Replace π with 3, r with 2, and h with 5.
≈ 60	Simplify.

The estimate of 60 is close to the answer of 68.4. So, the answer is reasonable.

Example 2 **The volume of a cylinder is 150 cubic inches. Find the height of the cylinder. Round to the nearest whole number.**



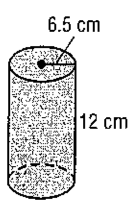
$V = \pi r^2 h$	Volume of a cylinder.
$150 = 3.14 \cdot 2^2 \cdot h$	Replace V with 150, π with 3.14, and r with 2.
$150 = 12.56h$	Simplify.
$12 \approx h$	Divide each side by 12.56. Round to the nearest whole number.

The height is about 12 inches.

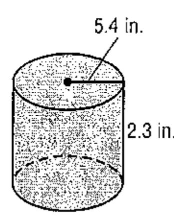
Exercises

Find the volume of each cylinder. Round to the nearest tenth.

1.

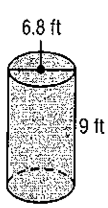


2.

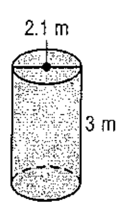


3. radius: 1.3 m
height: 3 m

4.



5.



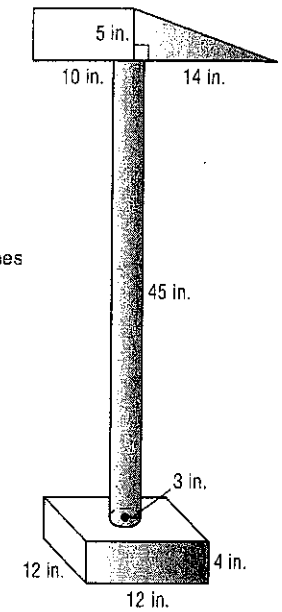
6. diameter: 11 cm
height: 6 cm

12-3 Study Guide and Intervention *(continued)*

Volume of Cylinders

Volumes of Composite Figures You can find the volume of composite figures with cylinders by separating the figure into the different pieces.

Example **PODIUMS A school principal ordered a podium for the debate club. Find the volume of the podium.**



The volume is the sum of the rectangular prism base, the cylindrical column, and the triangular prism top.

Step 1 Find the volume of the rectangular prism.

$V = Bh$ Volume of a prism

$V = 12 \cdot 12 \cdot 4$ The length and width are each 12 inches and the height is 4 inches

$= 576$ Simplify.

The volume of the rectangular prism base is 576 in^3.

Step 2 Find the volume of the cylinder.

$V = \pi r^2 h$ Volume of a cylinder

$V = 3.14 \cdot 3^2 \cdot 45$ Replace π with 3.14, r with 3, and h with 45.

≈ 1271.7 Simplify.

The volume of the cylinder is about 1271.7 in^3.

Step 3 Find the volume of the triangular prism.

$V = Bh$ Volume of a triangular prism

$V = \frac{1}{2} \cdot 14 \cdot 10 \cdot 5$ The length is 14, the width is 10, and the height is 5.

$= 350$ Simplify.

The volume of the triangular prism is 350 in^3.

Step 4 Find the volume of the composite figure.

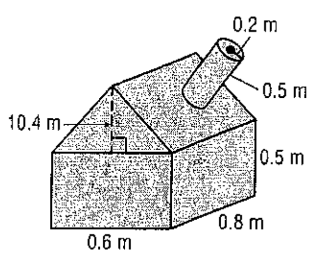
$576 + 1271.7 + 350 = 2197.7$

So, the total volume of the podium is 2197.7 in^3.

Exercises

Find the volume of each figure. Round to the nearest tenth.

1.



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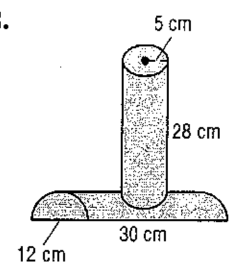

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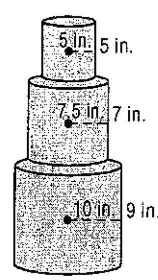


12-4 Study Guide and Intervention

Volume of Pyramids, Cones, and Spheres

Volume of a Pyramid A pyramid has $\frac{1}{3}$ the volume of a prism with the same base and height. To find the volume V of a pyramid, use the formula $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height of the pyramid.

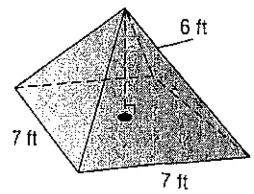
Example 1 **Find the volume of the pyramid.**

$V = \frac{1}{3}Bh$ Volume of a pyramid

$V = \frac{1}{3}(7 \cdot 7.6)$ The base is a square, so $B = 7 \cdot 7$. The height of the pyramid is 6 ft.

$V = 98$ Simplify.

The volume is 98 ft^3.



Volume of a Cone A cone has $\frac{1}{3}$ the volume of a cylinder with the same base and height. To find the volume V of a cone, use the formula $V = \frac{1}{3}\pi r^2h$, where r is the radius and h is the height of the cone.

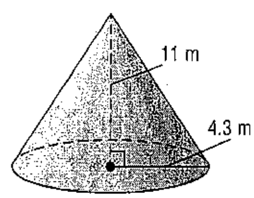
Example 2 **Find the volume of the cone. Round to the nearest tenth.**

$V = \frac{1}{3}\pi r^2 h$ Volume of a cone

$V = \frac{1}{3}\pi (4.3)^2 \cdot 11$ Replace r with 4.3 and h with 11.

$V \approx 213.0\ m^3$ Simplify. Round to the nearest tenth.

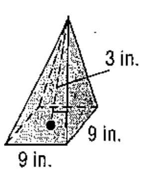
The volume is about 213.0 m^3.



Exercises

Find the volume of each figure. Round to the nearest tenth, if necessary.

1.



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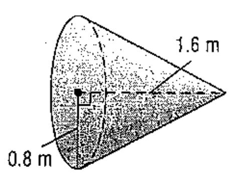

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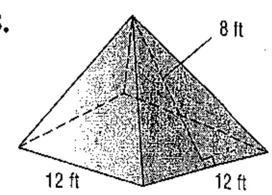


4. Square pyramid: length 1.2 centimeters, height 5 centimeters

5. Cone: diameter 4 yards, height 7 yards

6. Rectangular prism: length 14.5 meters, width 5.2 meters, height 6.1 meters

12-4 Study Guide and Intervention *(continued)*

Volume of Pyramids, Cones, and Spheres

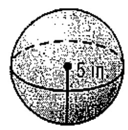
Volume of a Sphere To find the volume V of a sphere, use the formula $V = \frac{4}{3}\pi r^3$, where r is the radius.

Example 1 **Find the volume of the sphere. Round to the nearest tenth.**

$V = \frac{4}{3}\pi r^3$ Volume of a sphere

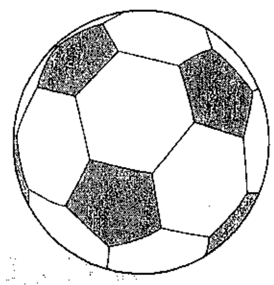
$V = \frac{4}{3}\pi (5)^3$ Replace r with 5.

$V \approx 523.6 \text{ in}^3$ Simplify.



The volume is about 523.6 in^3.

Example 2 **SOCCER A giant soccer ball has a diameter of 40 inches. Find the volume of the soccer ball. Then find how long it will take the ball to deflate if it leaks at a rate of 100 cubic inches per hour.**



Understand You know the diameter of the soccer ball. You know the rate at which it is losing air.

Plan Find the volume of the ball. Find how long it will take to deflate.

Solve $V = \frac{4}{3}\pi r^3$ Volume of a sphere

$= \frac{4}{3}\pi \cdot 20^3$ Since $d = 40$, replace r with 20.

$\approx 33{,}493.3 \text{ in}^3$ Simplify.

Use a proportion.

$$\frac{100 \text{ in}^3}{1 \text{ hour}} = \frac{33{,}493.3 \text{ in}^3}{x \text{ hour}}$$

$$100x = 33{,}493.3$$

$$x \approx 334.9$$

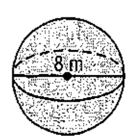
So, it will take approximately 335 hours for the ball to deflate.

Exercises

Find the volume of each sphere. Round to the nearest tenth.

1.


2.


3.


4. Sphere: radius 5.2 miles

5. Sphere: diameter 11.6 feet

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12-5 Study Guide and Intervention

Surface Area of Prisms

Lateral Area and Surface Area A prism consists of two parallel, congruent bases and a number of non-base faces. The non-base faces are called **lateral faces**. The **lateral area** of a figure is the sum of the areas of the lateral faces. The **surface area** of a figure is the total area of all the faces, or the sum of the lateral area plus the area of the bases.

To find the lateral area L of a prism with a height h and base with a perimeter P, use the formula $L = Ph$.

To find the surface area S of a prism with a lateral area L and a base area B, use the formula $S = L + 2B$. This can also be written as $S = Ph + 2B$.

Example 1 **Find the lateral and surface area of the rectangular prism.**

a. Find the lateral area.

$L = Ph$

$L = (2\ell + 2w)h$

$= (2 \cdot 2.1 + 2 \cdot 2.8)5.8$

$= 56.84 \text{ ft}^2$

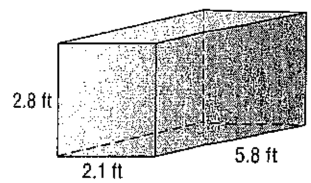
b. Find the surface area.

$S = L + 2B$

$S = L + 2\ell w$

$= 56.84 + 2 \cdot 2.1 \cdot 2.8$

$= 68.6 \text{ ft}^2$



Example 2 **Find the lateral and surface area of the triangular prism.**

a. Find the lateral area.

$L = Ph$

$= (5 + 5 + 6)7$

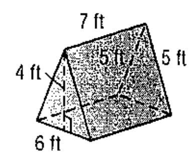
$= 112 \text{ ft}^2$

b. Find the surface area.

$S = L + 2B$

$S = 112 + 2 \cdot \frac{1}{2} \cdot 6 \cdot 4$

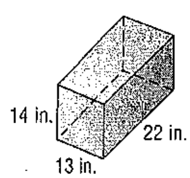
$= 136 \text{ ft}^2$



Exercises

Find the lateral and surface area of each prism. Round to the nearest tenth, if necessary.

1.



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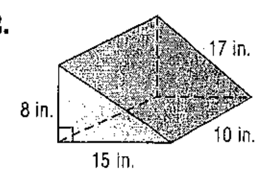

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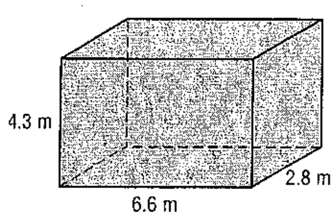

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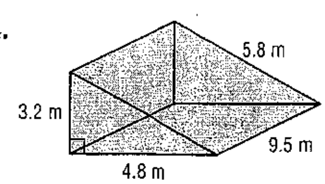


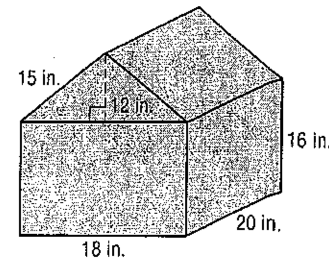
5. Cube: side length 8.3 centimeters

12-5 Study Guide and Intervention (continued)

Surface Area of Prisms

Problem Solving You can apply the formulas for lateral area and surface area to solve problems.

Example **CRAFTS Lena built a house out of cardboard. The roof is a triangular prism and the main part of the house is a rectangular prism. She wants to paint both parts before gluing them together. Find the amount of paint Lena needs if 1 ounce covers about 400 square inches.**



Triangular prism

a. Find the lateral area.

$L = Ph$
$= (15 + 15 + 18)20$
$= 960 \text{ in}^2$

b. Find the surface area.

$S = L + 2B$
$S = 960 + 2 \cdot \frac{1}{2} \cdot 18 \cdot 12$
$= 1176 \text{ in}^2$

Rectangular prism

a. Find the lateral area.

$L = Ph$
$L = (2\ell + 2w)h$
$= (2 \cdot 18 + 2 \cdot 16)20$
$= 1360 \text{ in}^2$

b. Find the surface area.

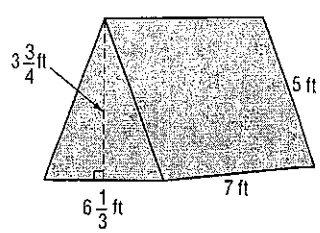
$S = L + 2B$
$S = L + 2\ell w$
$= 1360 + 2 \cdot 18 \cdot 16$
$= 1936 \text{ in}^2$

So, or the total area to be painted is 1176 + 1936 or 3112 in^2.
Since 3,112 ÷ 400 ≈ 7.75, Lena will need about 8 ounces of paint.

Exercises

1. **PAINTING** The walls of the school gym are being repainted. The gym is 50 feet long, 25 feet wide, and 16 feet high. Each wall will receive 2 coats of paint. If one gallon of paint covers 400 square feet, how many gallons are required?

2. **SPRAY-PAINTING** Kayla bought the tent shown at the right. She wants to spray all surfaces of the tent with waterproofing spray. Each 10-ounce bottle of spray will cover about 35 square feet. How many bottles of spray does Kayla need?



3. **PARTY FAVORS** For her birthday party, Rayna bought 12 boxes to decorate and give as party favors. She wants to decorate the boxes by covering them in fabric. Each box is a cube with side lengths of 5 inches. How many square inches of fabric does Rayna need?

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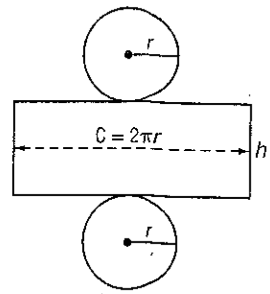
12-6 Study Guide and Intervention

Surface Area of Cylinders

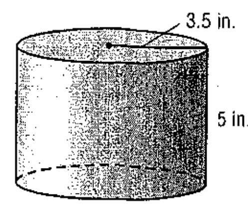
Surface Area of Cylinders As with a prism, the surface area of a cylinder is the sum of the lateral area and the area of the two bases. If you unroll a cylinder, its net is a rectangle (lateral area) and two circles (bases).



The lateral area L of a cylinder with radius r and height h is the product of the circumference of the base ($2\pi r$) and the height h. This can be expressed by the formula $L = 2\pi rh$.

The surface area S of a cylinder with a lateral area L and a base area B is the sum of the lateral area and the area of the two bases. This can be expressed by the formula $S = L + 2B$ or $S = 2\pi rh + 2\pi r^2$.

Example **Find the lateral and surface area of the cylinder.**



a. Find the lateral area.

$L = 2\pi rh$

$= 2 \cdot \pi \cdot 3.5 \cdot 5$

$= 35\pi \text{ in}^2$ exact answer

$\approx 109.9 \text{ in}^2$ approximate answer

b. Find the surface area.

$S = L + 2\pi r^2$

$= 35\pi + 2\pi(3.5)^2$

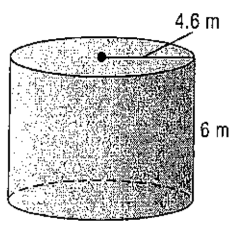
$= 59.5\pi \text{ in}^2$

$\approx 186.8 \text{ in}^2$

Exercises

Find the lateral and surface area of each cylinder. Round to the nearest tenth.

1.



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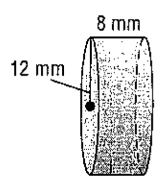

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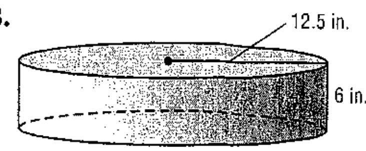

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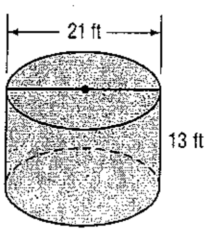

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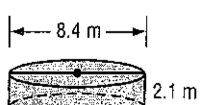

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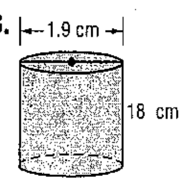


7. diameter of 20 yards and a height of 22 yards

8. radius of 7.6 centimeters and a height of 10.2 centimeters

12-6 Study Guide and Intervention *(continued)*

Surface Area of Cylinders

Problem Solving You can apply the formulas for lateral area and surface area to solve problems involving comparisons.

Example **DESIGN Marc studied package design in art class. He designed two cylindrical packages. One has a height of 4 inches and a diameter of 2.5 inches. The other has a height of 2.5 inches and a diameter of 4 inches. Which package has the greatest lateral area? Which has the greatest surface area?**

Step 1 Find the lateral area of both packages.

Lateral area of Package A

$L = 2\pi rh$
$= 2 \cdot \pi \cdot 1.25 \cdot 4$
$= 10\pi \text{ in}^2$
$\approx 31.4 \text{ in}^2$

Lateral area of Package B

$L = 2\pi rh$
$= 2 \cdot \pi \cdot 2 \cdot 2.5$
$= 10\pi \text{ in}^2$
$\approx 31.4 \text{ in}^2$

The lateral areas of the two packages are the same.

Step 2 Find the surface area of both packages.

Surface area of Package A

$S = L + 2\pi r^2$
$= 10\pi + 2\pi(1.25)^2$
$= 13.125\pi \text{ in}^2$
$\approx 41.2 \text{ in}^2$

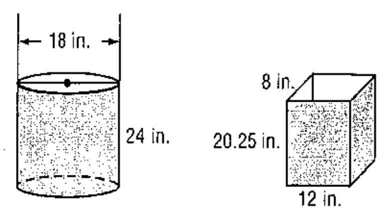
Surface area of Package B

$S = L + 2\pi r^2$
$= 10\pi + 2\pi(2)^2$
$= 18\pi \text{ in}^2$
$\approx 56.5 \text{ in}^2$

The surface area of Package B is greater than the surface area of Package A.

Exercises

1. **PAINTING** Gina is painting the garbage cans shown at the right. Both cans have the same volume. Which can has the greatest surface area? Explain.



2. **INSULATION** James is wrapping pipes in insulation. One pipe has a radius of 1.5 inches and a length of 30 inches. The other pipe has a radius of 3 inches and a length of 12.5 inches. Which pipe needs more insulation? Explain.

3. **STORAGE** There are two large cylindrical storage tanks at a factory. Both tanks are 12 feet high. One tank has a diameter of 8 feet and the other has a diameter of 16 feet. How does the surface area of the smaller tank relate to the surface area of the larger tank?

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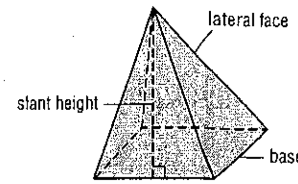
12-7 Study Guide and Intervention

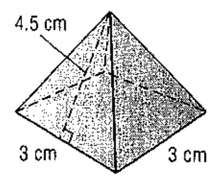
Surface Area of Pyramids and Cones

Surface Area of Pyramids **Regular pyramids** have bases which are a regular polygon and lateral faces which are congruent isosceles triangles. The height of each lateral face is called the **slant height** of the pyramid.



The lateral area L of a regular pyramid is half the perimeter P of the base times the slant height ℓ or $L = \frac{1}{2}P\ell$. The total surface area S of a regular pyramid is the lateral area L plus the area of the base B or $S = L + B$, or $S = \frac{1}{2}P\ell + B$.

Example **Find the lateral and total surface area of the square pyramid.**



a. Find the lateral area.

$L = \frac{1}{2}P\ell$	Write the formula.
$L = \frac{1}{2}(3 \cdot 4)4.5$	Replace P with $3 \cdot 4$ and ℓ with 4.5.
$= 27\ \text{cm}^2$	Simplify.

b. Find the surface area.

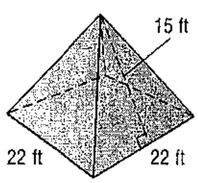
$S = L + B$	Write the formula.
$S = 27 + (3 \cdot 3)$	Replace L with 27 and B with $3 \cdot 3$.
$= 36\ \text{cm}^2$	Simplify.

The lateral surface area is 27 cm^2, and the total surface area is 36 cm^2.

Exercises

Find the lateral and surface area of each regular pyramid. Round to the nearest tenth, if necessary.

1.



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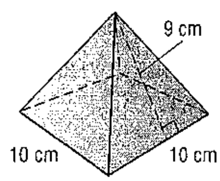

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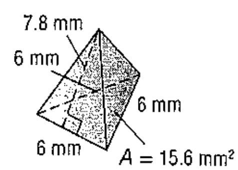

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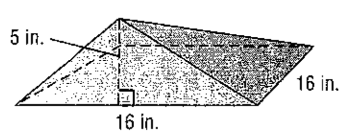

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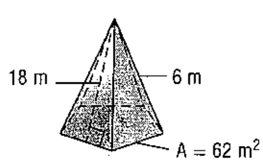

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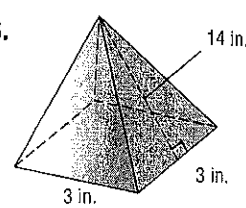

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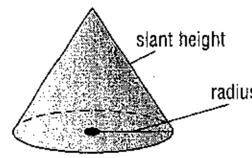
12-7 Study Guide and Intervention *(continued)*

Surface Area of Pyramids and Cones

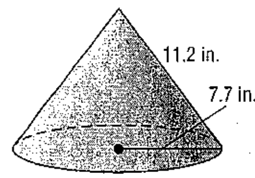
Surface Area of Cones

The lateral area L of a cone is the product of π, the radius r, and the slant height ℓ. This can be represented by the formula $L = \pi r\ell$.

The surface area S of a cone is the lateral area L plus the area of the base or πr^2. This can be represented by the formula $S = L + \pi r^2$.



Example **Find the lateral and total surface area of the cone. Round to the nearest tenth, if necessary.**



a. Find the lateral area.

$L = \pi r\ell$	Write the formula.
$L = \pi(7.7)(11.2)$	Replace r with 7.7 and ℓ with 11.2.
$\approx 270.8 \text{ in}^2$	Simplify.

b. Find the surface area.

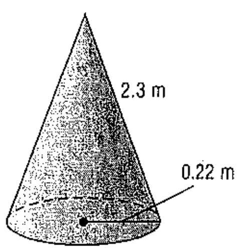
$S = L + \pi r^2$	Write the formula.
$S = 270.8 + \pi(7.7)^2$	Replace r with 7.7.
$\approx 457 \text{ in}^2$	Simplify.

The surface area is about 457 square inches.

Exercises

Find the lateral and surface area of each cone. Round to the nearest tenth, if necessary.

1.



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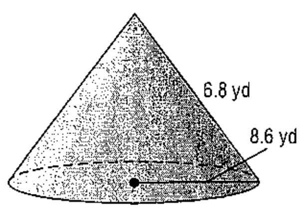

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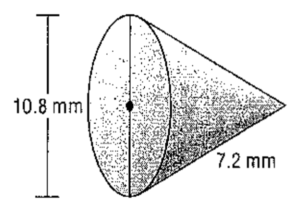

4. Cone: radius 7.2 meters, slant height 12 meters

5. Cone: diameter 16 inches, slant height 9 inches

6. Cone: diameter 5.5 yards, slant height 10 yards

7. Cone: diameter 3.6 feet, slant height 5.1 feet